Handbook of Filter Synthesis

Anatol I. Zverev

Consulting Engineer

Westinghouse Electric Corporation

John Wiley and Sons, Inc. *New York • London • Sydney*

9

Helical Filters

9.1 INTRODUCTION

In the VHF range, high-selectivity filters cannot be realized by conventional techniques. All filters require certain minimum values for the quality factor of their resonant circuits. At frequencies above 30 Mc, high-quality lumped elements, inductances, and capacitances do not exist. The piezoelectric crystal resonator, although of high quality, is not very flexible in realization of filters with wide bandwidths. Finally, the coaxial resonator cannot be used in the VHF range because of its large size. However, in the UHF range the cavity resonator (shown in Fig. 9.1) with tuning adjustment and coupling loops is very useful.

The toroidal inductor is not generally used at frequencies above 30 Mc because of its high distributed capacitance. The single-layer solenoid can be used to a degree, but the maximum Q realizable is about 200. Also, in filter applications the coils must be isolated from each other, requiring the use of shields, which, when placed in close proximity to the coils, reduces their Q. The quality factor can be improved by increasing the size of the solenoid, but the resulting filter is unproportionally large in comparison with the size of other components used in modern circuitry.

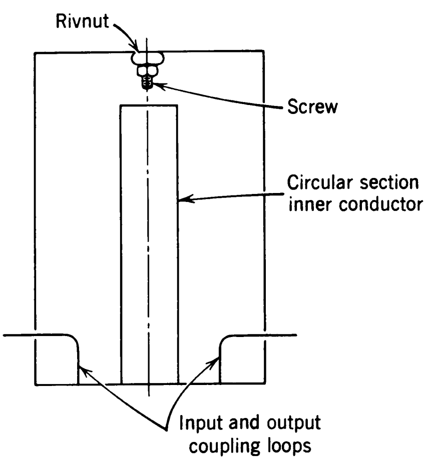


Fig. 9.1. Coaxial cavity for use in UHF filtering.

The piezoelectric crystal, although of high Q and small size, becomes impractical at higher frequencies. It is practically impossible to construct crystals whose fundamental frequency is above approximately 35 Mc since their physical dimensions become so small that good quality crystals cannot be produced. Harmonic crystals are generally utilized at frequencies above 30 Mc. The crystal is a high-quality device, but it has certain performance weaknesses, such as an unpredictable amount of spurious modes above the fundamental frequency (or any harmonic frequency), which discourages its use except for very low-percentage bandwidths (below 1%). These shortcomings reduce its value for use in filter construction. Any attempt to create wideband crystal filters meets unsurpassable difficulties because of the necessity to include lossy spreading coils, which reduce the obtainable bandwidth from its maximum value. A crystal filter is basically a very-narrow-bandpass element.

For a long time, the filter-design engineer has looked for a new type of resonator which could be used to produce selective, lowloss filters in the VHF domain. Coaxial lines with helical inner conductors have been used in traveling-wave tubes, parametric amplifiers, high-Q resonators, high characteristic-impedance transmission lines, low-frequency antenna designs, and in impedance-matching techniques for frequencies as low as 300 kc. The use of helical resonators for filtering in the VHF–UHF range will be discussed in this chapter.

9.2 HELICAL RESONATORS

Helical resonators of practical size and form factor and with high Q (of the order of 1000) can be constructed for the VHF and UHF ranges. Basically they resemble a coaxial quarter-wave resonator, except that the inner conductor is in the form of a single-layer solenoid, or helix. The helix is enclosed in a highly conductive shield of either circular or square cross-section. One lead of the helical winding is connected directly to the shield and the other end is open circuited.

As an example of space saving and a superior form factor, consider a coaxial resonator at 54 Mc with an unloaded Q of 550. The coaxial resonator would be 4.5 ft long by 0.7 in. in diameter. The same quality helical resonator would be 2 in. long by 1.5 in. in diameter.

Figure 9.2 is a sketch of the resonator with a circular cross-section. With these notations, the following set of equations can be given:

$$L = 0.025n^2d^2[1 - d/D^2]\ \mu\text{H per axial inch} \qquad (9.2.1)$$

where

$L =$ the equivalent inductance of the resonator in μH per axial inch.

$d =$ the mean diameter of the turns in inches.

$D =$ the inside diameter of the shield in inches.

$n = 1/\tau =$ turns per inch, where τ is the pitch of the winding in inches. (9.2.2)

Empirically for an air dielectric,

$$C = \frac{0.75}{\log_{10}(D/d)}\ \mu\mu\text{F per axial inch} \qquad (9.2.3)$$

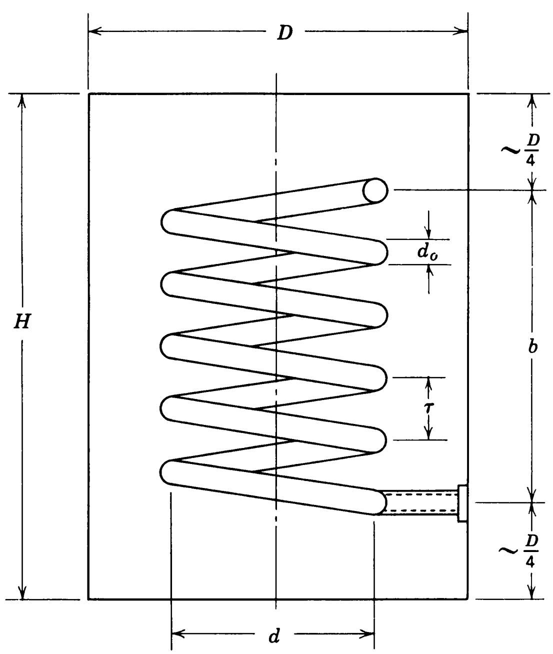


Fig. 9.2. Helical resonator with circular cross section.

This equation is valid only for the following condition,

$$\frac{b}{d} = 1.5 \qquad (9.2.4)$$

where b is the axial length of the coil in inches.

These equations and all those below are accurate for the resonator when it is realized between the following limits:

$$1.0 < \frac{b}{d} < 4.0$$

$$0.45 < \frac{d}{D} < 0.6$$

$$0.4 < \frac{d_0}{\tau} < 0.6 \text{ at } \frac{b}{d} = 1.5$$

$$0.5 < \frac{d_0}{\tau} < 0.7 \text{ at } \frac{b}{d} = 4.0$$

$$\tau < \frac{d}{2}$$

where d_0 is the diameter of the conductor in inches.

The axial length of the coil is approximately equivalent to a quarter wavelength. This actual length is much shorter than the free-space length, which is given by the expression,

$$\frac{\lambda}{4} = \frac{c}{4f_0}10^{-6} \qquad (9.2.5)$$

where c is the speed of light in free-space and f_0 is the operating frequency in megacycles per second. The actual length of the coil in inches can be expressed by the following equation:

$$b = \frac{250}{f_0\sqrt{LC}} \qquad (9.2.6)$$

where f_0 is the resonant frequency in megacycles per second.

This expression is based on theoretical considerations, but a working equation can be formulated with the help of the following expression.

$$\text{wave velocity } v = f_0\lambda = \frac{2\pi \text{ rad}}{2\pi\sqrt{LC}} = \frac{1000}{\sqrt{LC}} \qquad (9.2.7)$$

Because of the fringe effect and self capacitance of the coil, the electrical length of the coil is approximately 6% less than a quarter wavelength. The empirical value of b is reduced by 6% and is given below.

$$b = \frac{0.94\lambda}{4} = \frac{0.235v}{f_0} = \frac{235}{f_0\sqrt{LC}} \qquad (9.2.8)$$

The number of turns per inch is obtained by substituting Eqs. 9.2.1 and 9.2.3 into Eq. 9.2.8.

$$\frac{1}{\tau} = n = \frac{1720}{f_0 bd}\left[\frac{\log_{10}(D/d)}{1-(d/D)^2}\right]^{1/2} \text{ turns per inch} \quad (9.2.9)$$

The total number of turns N is given by

$$N = nb = \frac{1720}{f_0 D(d/D)}\left[\frac{\log_{10}(D/d)}{1-(d/D)^2}\right]^{1/2} \text{ turns} \quad (9.2.10)$$

The characteristic impedance of the resonator is expressed by

$$Z_0 = 1000\sqrt{\frac{L}{C}} = 183nd\left[\left(1-\frac{d}{D}\right)^2 \log_{10}\frac{D}{d}\right]^{1/2} \text{ ohms} \quad (9.2.11)$$

$$\frac{d}{D} = 0.55 \text{ and } \frac{b}{d} = 1.5, \quad \text{then } N = \frac{1900}{f_0 D} \text{ turns} \quad (9.2.12)$$

and

$$Z_0 = \frac{98000}{f_0 D} \text{ ohms} \quad (9.2.13)$$

If the shield is of square cross section, the following equations are applicable:

$$S = \text{length of one side of the square} \approx D/1.2 \quad (9.2.14)$$

$$Q = 60S\sqrt{f_0} \quad (9.2.15)$$

$$N = \frac{1600}{f_0 S} \quad (9.2.16)$$

$$n = \frac{1}{\tau} = \frac{1600}{S^2 f_0} \quad (9.2.17)$$

$$Z_0 = \frac{81500}{f_0 S} \quad (9.2.18)$$

$$d = 0.66S \text{ for } \frac{d}{D} = 0.55 \quad (9.2.19)$$

$$b = S \text{ for } \frac{b}{d} = 1.5 \quad (9.2.20)$$

$$H = 1.6S \quad (9.2.21)$$

Figure 9.3 shows a nomogram constructed from Eqs. 9.2.14 to 9.2.21. This nomogram is to be used for helical resonators in shields of a square cross section, the resonator that physically lends itself best to filter design.

Quality Factor

The helical resonator has solved the problem of high-quality resonators in the VHF range. In a reasonable volume, they provide a tuned circuit whose Q is higher than the normal lumped circuit. Possible causes of dissipation in the resonator are losses in the conductor, in the shield, and in the dielectric. The Q of a resonator is defined by

$$Q = 2\pi f \frac{\text{energy stored}}{\text{power dissipated}} \quad (9.2.22)$$

Losses in the helical resonator include the actual loss in the helix, a copper loss as influenced by the skin and the proximity effects. There is also an additional loss owing to currents in the shield. The resistance of the coil can be expressed as

$$R_c = \frac{n\pi\, d\phi\sqrt{f}}{12{,}000 d_0}$$

or,

$$R_c = \frac{0.083}{1000}\frac{\phi}{nd_0}n^2\pi\sqrt{f} \text{ ohms per axial inch} \quad (9.2.23)$$

An additional resistance due to the shield is given by

$$R_s = \frac{9.37n^2b^2(d/2)^4\sqrt{1.724f}}{b[D^2(b+d)/8]^{4/3}} \times \sqrt{\frac{\rho_s}{\rho_{cu}}} \times 10^{-4} \text{ ohms per axial inch} \quad (9.2.24)$$

The unloaded Q of a resonant line is given by

$$Q_u = \frac{\beta}{2\alpha} \quad (9.2.25)$$

If R_c and R_s are assumed in series, the Q of the resonant line is expressed as

$$Q_u = \frac{2\pi f_0 L}{R_s + R_c} \quad (9.2.26)$$

In this form, the dielectric losses are neglected. For a resonator with a copper coil and copper shield, Eqs. 9.2.23 and 9.2.24 can be substituted into Eq. 9.2.26 and the final expression for the unloaded Q is

$$Q_u = 220\frac{(d/D)-(d/D)^3}{1.5+(d/D)^3}D\sqrt{f_0} \quad (9.2.27)$$

$$Q_u \approx 50D\sqrt{f_0} = 60S\sqrt{f_0} \quad (9.2.28)$$

The simplified equation is accurate to $\pm 10\%$ and is derived with three practical limitations; when

$$0.45 < \frac{d}{D} < 0.6, \quad \frac{b}{d} > 1.0 \quad \text{and} \quad d_0 > 5\delta$$

where δ is the skin depth.

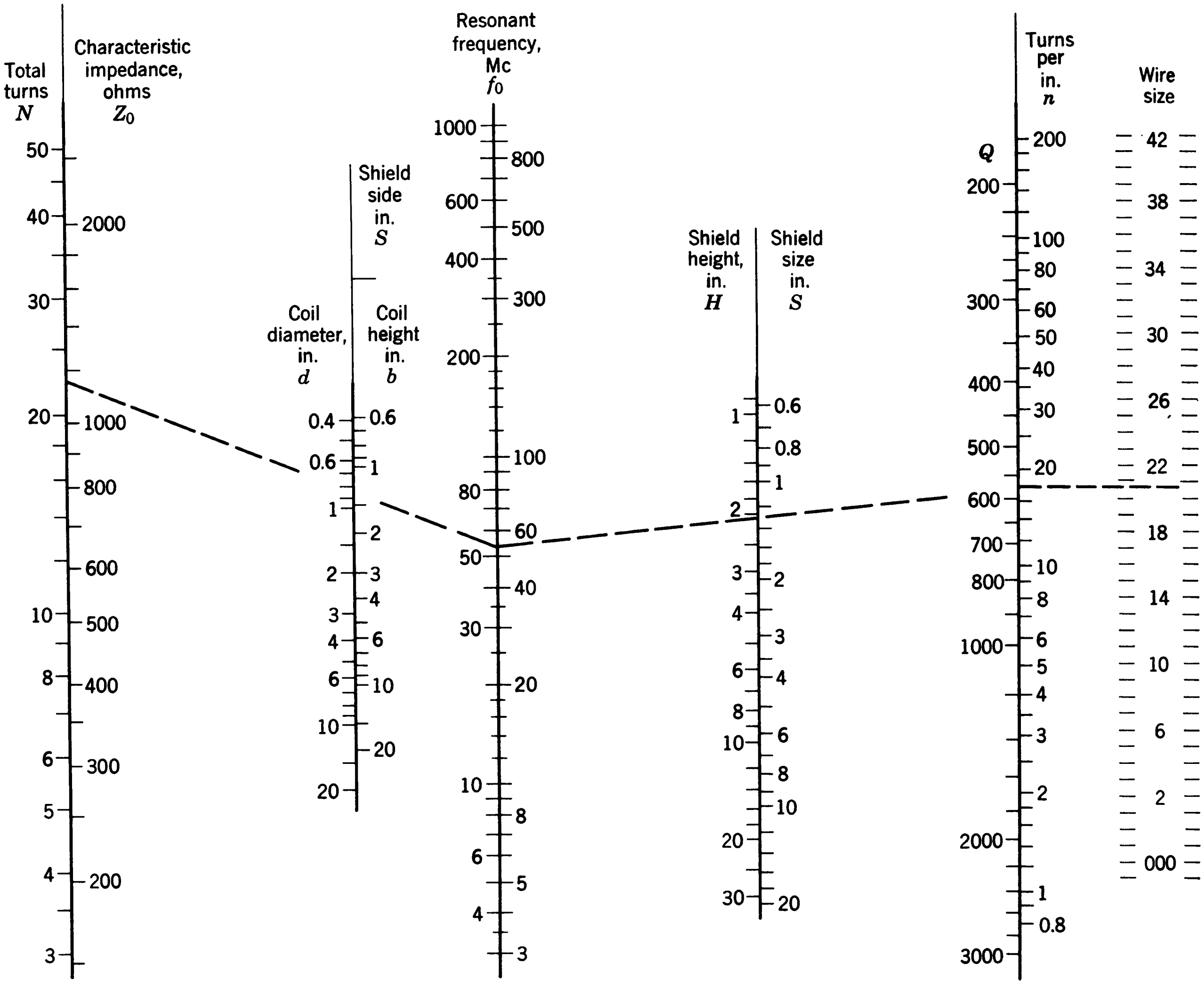


Fig. 9.3. Nomogram for helical resonators in shields of square cross section.

For copper conductors,

$$\delta = \frac{2.60 \times 10^{-3}}{\sqrt{f_0}} \text{ inch} \tag{9.2.29}$$

To show how important the volume of the resonator is, the Q_u as a function of volume is

$$Q_u = 50 \sqrt[3]{\text{Vol}} \sqrt{f_0}, \tag{9.2.30}$$

when

$$0.4 < \frac{d}{D} < 0.6 \text{ and } 1 < \frac{b}{d} < 3.$$

Figure 9.4 illustrates how rapidly the unloaded Q decreases as a function of d/D and how important it is to keep this ratio between the specified limits.

If the condition $d_0 > 5\delta$ is not fulfilled, the Q of the resonator will be lower than that predicted by Eq. 9.2.28. For $d_0/\tau = 0.5$, the wire diameter d_0 is given by

$$d_0 = \frac{dDf_0}{3{,}800} \text{ inches} \tag{9.2.31}$$

In a given space, d_0 can only be increased if the number of turns is reduced. This means that the inductance must be reduced. Thus for a fixed frequency, the capacitance must be increased.

Measurement of Resonator Q

The problem of finding the unloaded Q of the resonator is not easy. Many methods have been proposed but most have been inconvenient or impractical. However, the unloaded Q can be estimated quite accurately from the loaded Q and the insertion loss. This relation between the insertion loss and Q

when generator and load impedances are equal, is

$$L_{\text{dB}} = 20 \log \frac{U}{U - 1} \tag{9.2.32}$$

where

$$U = \frac{Q_{\text{unloaded}}}{Q_{\text{minimum}}}$$

In this case, $Q_{\min}$ is the loaded Q determined from the relation

$$Q_{\min} = \frac{f_0}{\text{BW}_{3\,\text{dB}}}$$

where $\text{BW}_{3\,\text{dB}}$ is the measured 3-dB bandwidth. When loop coupling is used into and out of the resonator, the insertion loss, and hence $Q_{\min}$, will be a function of the coupling between loops and the losses in the loop circuits. It is desirable to use very loose coupling in order that the effect of coupling between loops may be neglected. Figure 9.5 gives a plot of Eq. 9.2.32. The insertion loss is measured by the substitution method when all coaxial cables are as short as possible. The value of unloaded Q is evaluated by multiplying the value of $Q_{\min}$ by the value of U which corresponds to the measured insertion loss. If the insertion loss of the resonator is greater than 25 dB, the correction factor for the unloaded Q will be 1.05 or less. At this condition, $Q_{\min}$ will only be in error of Q_{unl} by 5% and it is self evident that Q_{unl} could be calculated from

$$\frac{f_0}{\text{BW}_{3\,\text{dB}}}$$

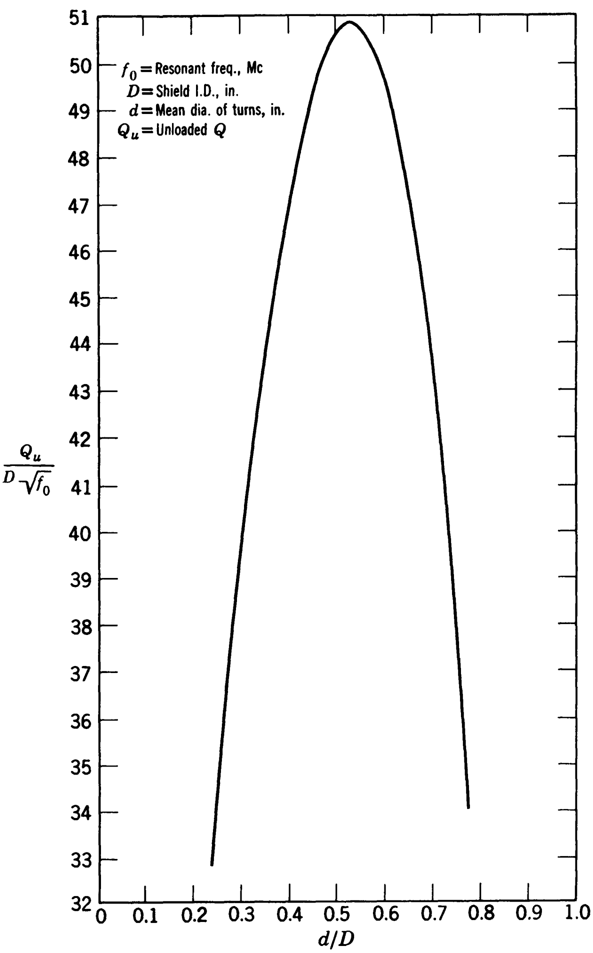


Fig. 9.4. Unloaded Q of the helical resonator.

Physical Construction of Resonator

To obtain the predicted unloaded Q, several points for construction of the resonator should be remembered. The shield can be cylindrical, rectangular, or any other shape, but for simplicity of calculations, only the shields of circular and square cross sections have been considered. Any seams in the shield parallel to the coil axis should have good physical and electrical connection. Dip-brazed cans have been used extensively. If the coil end is run to the bottom cover of the shield, the cover must be solidly connected to the shield to reduce the losses and ensure the high-quality factor of the resonator. This connection may be done best by soldering, but in actual practice, the use of screws every few inches is permissible.

The length of the shield must be extended beyond the coil on each side by approximately one-quarter of the shield diameter, or for a square shield, by approximately 0.3 times the side of the square. If the coil were carried to the bottom of the resonator can without having this clearance, the lower few turns would be ineffective for storage of energy but would still contribute loss. The clearance at the top of the resonator is to reduce capacitive loading. Actually, the resonator could be built without top and bottom covers, since they have little effect upon the frequency and Q. However, the external field is greatly reduced by the use of these covers.

At the open end of the helix a high voltage exists. The coil should end smoothly, without sharp edges, and should not turn into or out of the helix. The coil form supporting the helix must be made of a low-loss material; otherwise, the quality factor of the resonator will be degraded seriously. Polystyrene rods have been used for this application in many helical filter designs. This material is known for its low-dissipation factor, making it electrically suitable. Two mechanical properties of polystyrene, however, limit its usefulness.

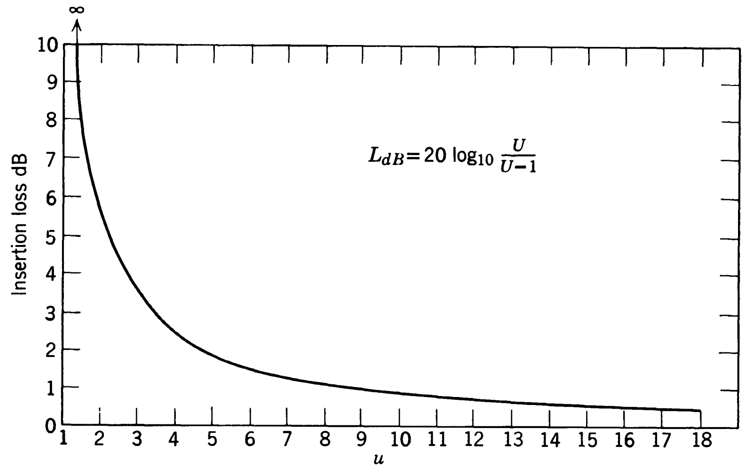


Fig. 9.5. General curve for insertion loss.

First, the softening temperature of polystyrene is relatively low, approximately 82°C. At this temperature distortion of the material occurs. Second, the coefficient of expansion of polystyrene is rather high, and special provisions must be made to achieve a filter which is temperature stable.

Temperature Compensation in Helical Filters

In variable temperature conditions, most of the components used for selective devices and coupling networks, exhibit appreciable change in losses as well as electrical values. For example, the quality factor of a coil decreases as the temperature increases, and the value of inductance usually increases with the temperature increase. Capacitors, such as the silver mica type also exhibit a similar effect, with a positive temperature coefficient.

Experience proves that temperature compensating devices only solve part of the problem, that of stabilizing resonant frequencies, since their presence in the circuit may deteriorate the quality factor of the original network. This is especially true in high-quality, high-frequency filters, where the midband insertion loss is increased due to their use. Since their loss factor is temperature dependent, filters employing temperature-compensating capacitors will still exhibit a varying insertion loss with temperature.

In the helical resonator filter, the polystyrene form on which the coil is wound expands with increasing temperature. This expansion is greater than the expansion of the wire itself, and also greater than the cavity expansion. The expansion of the form tends to force the coil to increase in diameter, and lowers the center frequency of the filter. By using the coil form of Fig. 9.6, this problem is almost entirely eliminated. Here, four slits running down the form absorb the expansion of the material, and the perimeter of the form will not change. Experiments have shown that the most sensitive part of the helix is the upper, unconnected end. It has been found that only

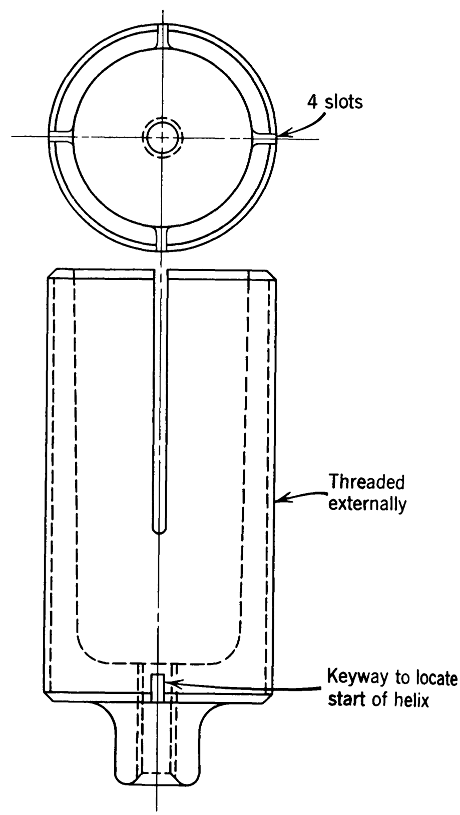


Fig. 9.6. Coil form used for helical resonator with temperature compensating feature.

approximately 60% of the total length of the coil form need be slotted to absorb the variation.

When insertion loss variation is critical, the change can be completely cancelled by use of resistive pads with temperature-sensitive resistors. The pads may be of the L or π variety using one or more sensistors, the temperature-sensitive element, along with regular carbon resistors.

9.3 FILTER WITH HELICAL RESONATORS

To develop a filter, it is first necessary to consider the required attenuation requirements, paying particular attention to the filters relative bandwidth, and its relationship to the minimum quality factor of the resonators necessary to realize the design.

For a given set of specifications, the value of the unloaded Q must exceed a certain $Q_{\min}$ for that filter to be realizable. Figure 9.7 shows the relationship between the required $q_{\min}$ [$Q_{\min} = q_{\min}(f_0/\Delta f)$] for a Butterworth and three different Chebyshev filters.

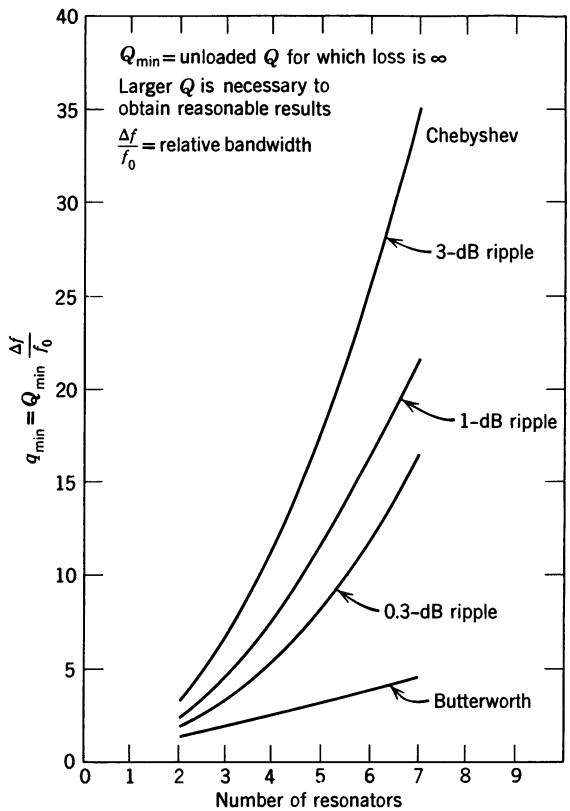


Fig. 9.7. Relative minimum unloaded Q for Butterworth and Chebyshev filters.

Example 1

A seven-pole filter (seven resonators) possessing a Butterworth response requires a $q_{\min}$ of 4.6. For a Chebyshev response with a 1-dB ripple in the passband, $q_{\min} = 21.9$. If Q_{unl} were equal to $Q_{\min}$, the insertion loss would be infinite. It is self evident that the unloaded Q must exceed $Q_{\min}$.

For this example, assuming a 1% bandwidth, $Q_{\min}$ equals 460 for the Butterworth filter and 2190 for the Chebyshev. It must be remembered that if components whose unloaded Q is barely equal to $Q_{\min}$ are used, the response can be achieved, provided certain predistorted values of coupling coefficients are used, but an extremely large value of insertion loss will result.

When the unloaded Q is greater than $Q_{\min}$, the loss of the filter does not primarily depend on the number of sections, but is exclusively controlled by the ratio U, given in Eq. 9.2.32. Once the minimum value of unloaded Q is obtained, and the quality factor of available components is determined, the loss in the filter is almost completely defined and varies very little with the shape of the filter, the number of sections, the bandwidth, etc.

Fubini and Guillemin give a curve which shows the minimum insertion loss at midband of Butterworth filters plotted as a function of the ratio U. The following two conclusions can be made:

(1) For moderate losses, the curves are very close to each other.

(2) The curves for one- and two-section filters are exactly the same and are expressed by Eq. 9.2.32.

From the previous Example 1, of a seven-pole Butterworth filter, assume the available unloaded Q of each section is 3000. The value of U can be computed as follows:

$$U = \frac{Q_{\text{unl}}}{Q_{\min}} = \frac{3000}{460} = 6.52$$

From Eq. 9.2.32 or Fig. 9.5,

$$L = 20 \log \frac{6.52}{5.52} = 20 \times 0.072 = 1.44 \text{ dB}$$

As mentioned before, this equation is only valid for one and two sections. For the example with seven resonators, a correction factor must be used. Figure 9.8 plots this correction factor and shows that the loss is always greater as the number of sections is increased.

Since the number of sections is seven, the correction factor is 1.27 and the actual insertion loss at midband

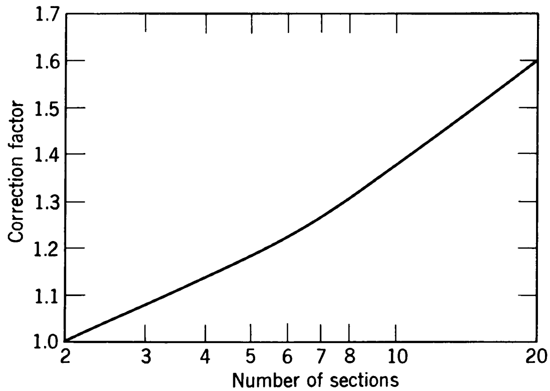


Fig. 9.8. Correction factor for insertion loss for more than two sections.

will be

$$L = 1.44 \times 1.27 = 1.83 \text{ dB}$$

It should be stated that this method of insertion loss is approximate, due to the empirical correction factor used. An exact method of calculation will be given later.

For a realizable Chebyshev filter, $q_{\min}$ is always higher than that of a Butterworth filter and may be found from Fig. 9.7. For the same unloaded Q and bandwidth, the insertion loss of a Chebyshev filter will be several times higher than that of a Butterworth filter.

Example 2

If the available unloaded Q is 5000 and the relative bandwidth is 1%, the insertion loss for a four-pole Butterworth filter will be 0.53 dB. For a Chebyshev filter with four poles (3-dB ripple) the expected insertion loss will be 2.67 dB. If the Q factor is only 3000, the former will result in 0.898 dB loss and the latter will exhibit 4.95 dB insertion loss. If the Q is 2000 the corresponding insertion losses will be 1.38 dB and 8.8 dB respectively.

Filter Construction

As previously stated, the only reason for using helical resonators is to reduce the size of the filter and to provide a low insertion loss in the passband. The design of a filter for a Butterworth and Chebyshev response is straightforward, and the evaluation of their circuit elements is well known. More interesting, however, are the equivalent schematics and the mechanical realization of the filter.

Even after carefully calculating the number of turns, and all dimensions of the resonator, it is very possible that the resonant frequency may be in error by as much as 10%. This must be adjusted without any distortion to the other dimensions so that the predicted Q will be obtained. This adjustment is made by using a tuning screw at the top of the helix. In the equivalent schematic the screw, because it is connected to ground, has the effect of providing capacitive loading for the helix.

Coupling into the Filter

The problem to be considered here is most important, namely, coupling into and out of the helical resonator filter. There are three methods of achieving this coupling: loop, probe, and tap coupling.

For loop coupling, a loop of approximately one turn is placed around the first helix. The loop is usually positioned slightly below the helix, in a plane perpendicular to the helix axis. The distance between the coupling loop and helix can be adjusted until a proper match is achieved between the filter and its load. The use of the loop generally yields relatively low impedance coupling and features a dc short to ground.

Loop coupling, although rather simple to use, has disadvantages. First, positioning of the loop so that the desired coupling requirements are satisfied is difficult. Second, supporting the loop so that shock and vibration requirements are satisfied is also difficult.

Next to be considered is probe coupling, in which a probe is placed close to the upper part of the helix. In this manner, no dc path is provided, and the coupling is mostly capacitive. A high impedance match can be achieved, and may be adjusted by varying the depth of penetration of the probe into the helical cavity.

The use of tap coupling has been found to be the most practical. This type of coupling has production advantages, as well as offering the necessary stability in order to satisfy shock and vibration specifications. A dc path is provided with tap coupling. The approximate position of the tap may be calculated, and the exact position then experimentally determined in the laboratory. Figure 9.9 shows how the tap is made.

The doubly loaded Q of the input and the output resonators are equal for the large Q case and given by

$$Q_d = Q_1 = Q_n = \frac{1}{2} q_1 \frac{f_0}{\mathrm{BW}_{3\,\mathrm{dB}}} \tag{9.3.1}$$

where

n = the number of resonators in the filter

Q_d = the doubly loaded Q

q_1 = normalized quality factor given in tables of 3-dB down k and q values in Chapter 6.

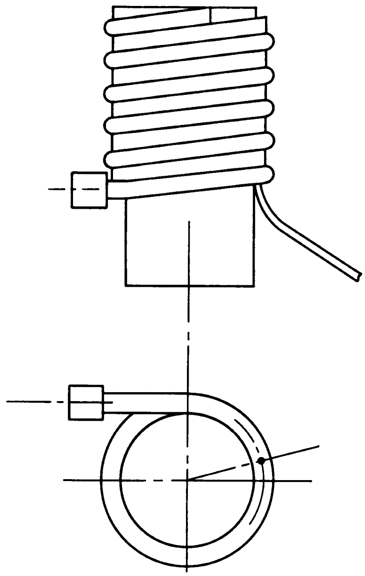


Fig. 9.9. Tap coupling.

For the Butterworth response, the list below may be referred to:

No. of Resonators	q_1
2	1.414
3	1.000
4	0.766
5	0.618
6	0.518
7	0.445

Applying transmission line theory, the following equation may be obtained.

$$\frac{R_b}{Z_0} = \frac{\pi}{4}\left(\frac{1}{Q_d} - \frac{1}{Q_u}\right) \tag{9.3.2}$$

which is divided equally between the generator and the second resonator when tapping the input coil. When tapping the output coil, R_b/Z_0 is divided equally between the load and the $(n - 1)$ resonator.

Also from resonant line theory,

$$\sin\theta = \sqrt{\frac{R_b}{2Z_0}\frac{R_{\text{tap}}}{Z_0}} \tag{9.3.3}$$

where θ is the electrical angle from the voltage standing wave minimum point (here, the helix ground). The tap is then placed $N\theta/90°$ turns from the grounded end of the helix.

Example 3

A helical filter exhibiting Butterworth characteristics is to be placed between the source impedance of 50 ohms and a load whose impedance is 1000 ohms. It is a four-section filter with a 2.5% bandwidth. Each resonator has 71.5 turns and has an unloaded Q of 300. The characteristic impedance of each resonator is 3630 ohms. Using Eqs. 9.3.1, 9.3.2 and 9.3.3:

$$Q_d = \frac{1}{2}0.766\frac{1}{0.025} = 15.32$$

$$\frac{R_b}{Z_0} = \frac{\pi}{4}\left(\frac{1}{15.32} - \frac{1}{300}\right) = 0.04865$$

For the 50-ohm source,

$$\sin\theta = \sqrt{0.02433\frac{50}{3630}} = 0.0183$$

$$\theta = 1.05°$$

$$\text{tap} = \frac{71.5 \times 1.05°}{90°} = 0.83 \text{ turns from grounded end}$$

For the 1000-ohm load,

$$\sin\theta = 0.0819$$

$$\theta = 4.70°$$

Tap = 3.73 turns from the grounded end.

For all three types of end coupling (that is, loop, probe, and tap), fine adjustment of the coupling from the outside of the filter is difficult. To add some flexibility to the filter, a form of fine coupling adjustment is often desirable. Coupling adjustment may be achieved by connecting a variable capacitor between the ground end of the helix and ground as in Fig. 9.10. Tap coupling is calculated as above for the approximate location of the tap. If the variable capacitor is large, the coupling will be unaffected, as if no capacitor were used, and the ground end of the resonator grounded. As the capacitance is decreased, the coupling into the resonator is decreased. This continues until no power is coupled in, which occurs when

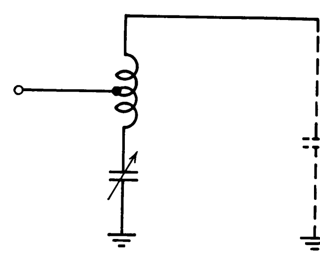


Fig. 9.10. Fine adjustment of tap coupling.

that portion of the helix below the tap and the capacitor is at series resonance. As the series capacitor is decreased further, coupling begins to increase. The fundamental frequency of the helix will shift slightly because of the supplementary capacitance, but the shift can be eliminated by the conventional method of tuning the helix.

This modification is especially useful for limited production, where the tap cannot be preset exactly and is of particular advantage when the source and load impedances are not known precisely. A dc short does not exist in this configuration.

Coupling Between Resonators

The coupling of helical resonators is considered the most complicated problem in the realization of a specific filter design. The problem arises because of the difficulty encountered in the mathematical analysis of the coupling.

Figure 9.11 defines the dimension h to be that part of one helix exposed to the adjacent helix. The shield may be either open at the bottom or the top of the resonator. The shield is made of the same material as the can and is solidly connected to the sides and top of the can (dip-brazed, or soldered). The dimension h determines the amount of coupling between resonators. The problem of accurately determining the dimension h for helical filters of all practical sizes and frequencies have not yet been solved. However, measurements taken on a coupling test block (Fig. 9.12) have yielded data that may be extended for use at frequencies in the 30-Mc range, with $S \approx 1.25$.

The test block is actually a two resonator filter, featuring a replaceable shield. The parameters necessary to calculate the coupling was measured, and the results were plotted (Fig. 9.13). With this test block, the shield thickness was $\frac{1}{32}$-inch. Most filters are made with $\frac{1}{16}$-inch thick shields. To obtain h for $\frac{1}{16}$-inch material, multiply the value of h obtained from the curve by 1.075.

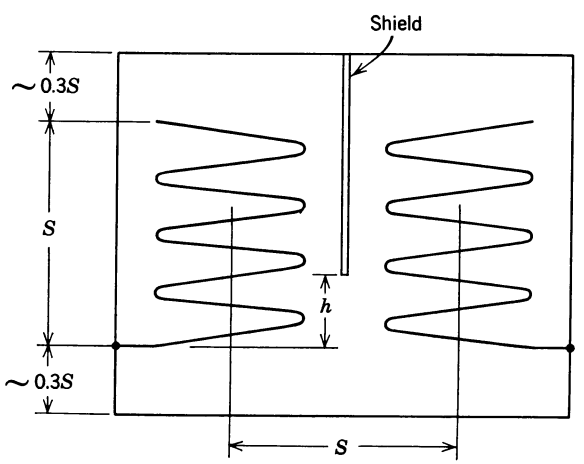


Fig. 9.11. Position of coupling shield between resonators.

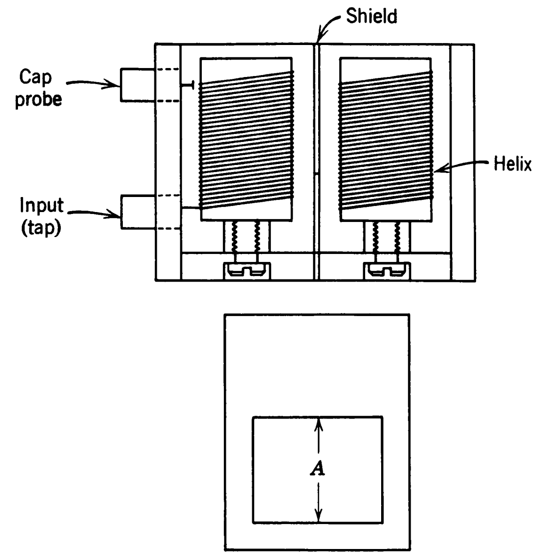


Fig. 9.12. Coupling test block.

The curve of Fig. 9.13 has been reduced to

$$K \times 10^{-3} = 0.071\left(\frac{h}{d}\right)^{1.91} \tag{9.3.4}$$

Use of this design method will yield results accurate to within approximately 6%.

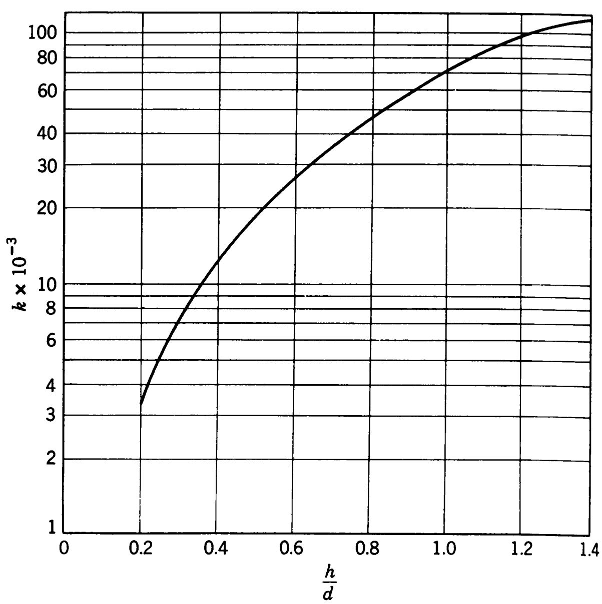


Fig. 9.13. Coupling between resonators as a function of h/d.

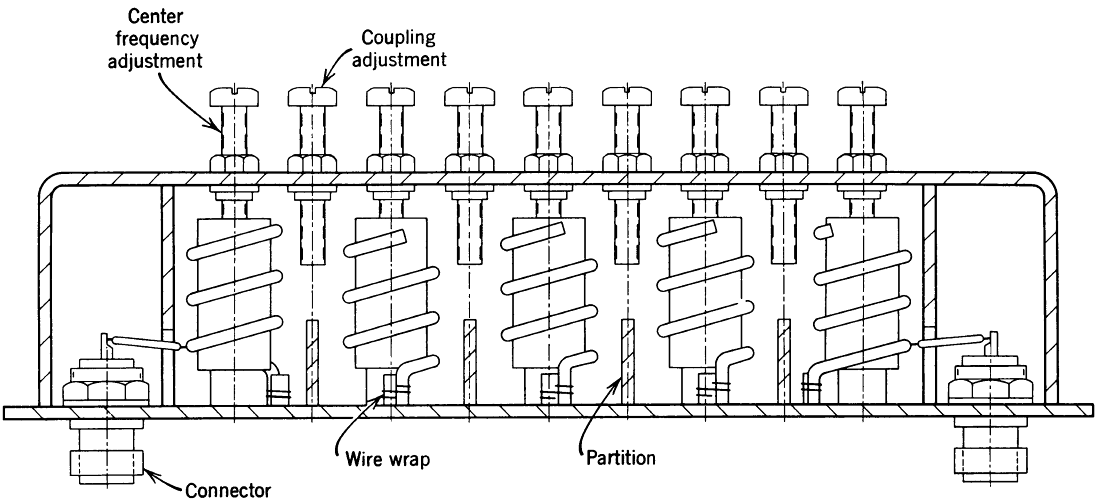


Fig. 9.14. 540-Mc bandpass helical filter with variable coupling feature.

Openings at the upper part of the resonator yield mostly capacitive coupling, and more attenuation is obtained on the lower side of the passband. Openings in the lower part of the partitions provide mostly inductive coupling, and the filter will exhibit more attenuation on the high-frequency side of the response curve.

When using capacitive coupling, a fine coupling adjustment can be obtained by using a screw inserted through the top of the can in line with the shield. Figure 9.14 shows a 540-Mc filter using this arrangement.

Determination of the Number of Resonators

Given any design problem, the first step is to determine the number of resonators necessary to fulfill the rejection requirement. The attenuation curves of Chapter 3 may be reviewed, or for Butterworth or Chebyshev filters, the nomographs of Figs. 5.1 and 5.2 may be used.

Example 4

A bandpass filter is required with the following specifications:

$$f_0 = 30 \text{ Mc}$$
$$BW_{3\,dB} = 1 \text{ Mc}$$
$$BW_{50\,dB} = 4 \text{ Mc}$$

It is desired that the response shape be Butterworth.

Using the attenuation curve for Butterworth filters in Chapter 3, and keeping in mind

$$\frac{BW}{BW_{3\,dB}} = \frac{4}{1} = 4$$

it can be seen by using four resonators, 48 dB is obtained at the frequency where 50-dB rejection is required. Going to five resonators, 60 dB is obtained.

Determination of Required Unloaded Q

To determine the size of the resonators required for the filter, we must first find the value of unloaded Q necessary to fulfill the insertion loss requirement. The method given earlier, in Examples 1 and 2, is only approximate. An exact method will be given here, making use of the quantities:

Q_u = the unloaded Q required

Q_L = the loaded $Q = f_0/BW_{3\,dB}$

$Q_0 = Q_u/Q_L$

q_1, q_n = the normalized Q of the first and last resonators

$k_{12} \cdots k_{(n-1)(n)}$ = the normalized coupling coefficients

The equations for insertion loss given in Table 9.1 are exact for any type response. The equations of Table 9.2 are for very low-loss Butterworth responses. For low-insertion losses, the higher order terms can be neglected.

Example 5

What unloaded Q is needed for a four-pole Butterworth filter with a 1 dB insertion loss, and a 3-dB bandwidth of 15 Mc at a center frequency of 500 Mc?

SOLUTION

$$IL = 20 \log \left[\frac{2.62}{Q_0^3} + \frac{3.41}{Q_0^2} + \frac{2.62}{Q_0} + 1\right]$$

for Q_0, neglecting the Q_0^3 term yields

$$Q_0 = 22.7$$

Table 9.1 Insertion Loss Equations for Any Type Response

$$n = 2 \qquad IL = 20 \log \left[\frac{\sqrt{q_n/q_1}}{k_{12}}\left(\frac{1}{Q_0} + \frac{1}{q_2}\right)\right]$$

$$n = 3 \qquad IL = 20 \log \left[\frac{\sqrt{q_n/q_1}}{k_{12}k_{23}}\left(\frac{1}{Q_0^2} + \frac{1}{q_n Q_0} + k_{23}^2\right)\right]$$

$$n = 4 \qquad IL = 20 \log \left\{\frac{\sqrt{q_n/q_1}}{k_{12}k_{23}k_{34}}\left[\frac{1}{Q_0^3} + \frac{1}{q_n Q_0^2} + \frac{(k_{23}^2 + k_{34}^2)}{Q_0} + \frac{k_{23}^2}{q_n}\right]\right\}$$

$$n = 5 \qquad IL = 20 \log \left\{\frac{\sqrt{q_n/q_1}}{k_{12}k_{23}k_{34}k_{45}}\left[\frac{1}{Q_0^4} + \frac{1}{q_n Q_0^3} + \frac{(k_{23}^2 + k_{34}^2 + k_{45}^2)}{Q_0^2} + \frac{(k_{23}^2 + k_{34}^2)}{q_n Q_0} + k_{23}^2 k_{45}^2\right]\right\}$$

$$n = 6 \qquad IL = 20 \log \left\{\frac{\sqrt{q_n/q_1}}{k_{12}k_{23}k_{34}k_{45}k_{56}}\left[\frac{1}{Q_0^5} + \frac{1}{q_n Q_0^4} + \frac{(k_{23}^2 + k_{34}^2 + k_{45}^2 + k_{56}^2)}{Q_0^3} + \frac{(k_{23}^2 + k_{34}^2 + k_{45}^2)}{q_n Q_0^2} + \frac{(k_{23}^2 k_{56}^2 + k_{23}^2 k_{45}^2 + k_{34}^2 k_{56}^2)}{Q_0} + \frac{k_{23}^2 k_{45}^2}{q_n}\right]\right\}$$

$$n = 7 \qquad IL = 20 \log \left\{\frac{\sqrt{q_n/q_1}}{k_{12}k_{23}k_{34}k_{45}k_{56}k_{67}}\left[\frac{1}{Q_0^6} + \frac{1}{q_n Q_0^5} + \frac{(k_{23}^2 + k_{34}^2 + k_{45}^2 + k_{56}^2 + k_{67}^2)}{Q_0^4} + \frac{(k_{23}^2 + k_{34}^2 + k_{45}^2 + k_{56}^2)}{q_n Q_0^3} + \frac{(k_{23}^2 k_{56}^2 + k_{23}^2 k_{67}^2 + k_{23}^2 k_{45}^2 + k_{34}^2 k_{56}^2 + k_{34}^2 k_{67}^2 + k_{45}^2 k_{67}^2)}{Q_0^2} + \frac{(k_{23}^2 k_{45}^2 + k_{23}^2 k_{56}^2 + k_{34}^2 k_{56}^2)}{q_n Q_0} + k_{23}^2 k_{45}^2 k_{67}^2\right]\right\}$$

Then

$$Q_u = Q_L Q_0 = \frac{f_0}{\text{BW}_{3\,\text{dB}}} Q_0$$

$$Q_u = 756$$

Predistorted k and q Values

The values of k and q given for the infinite q case in Chapter 6 can be used provided

$$Q_u \geq \left(\frac{f_0}{\text{BW}_{3\,\text{dB}}}\right) q_{\text{min}} = Q_{\text{min}}$$

As mentioned before, if $Q_u = Q_{\text{min}}$, an infinite midband insertion loss will result. If Q_u is only slightly larger than Q_{min}, the resulting filter will have a high value of insertion loss, and the 3-dB bandwidth of the resulting filter will be narrower than the design bandwidth. Only when

$$Q_u \approx 10 Q_{\text{min}}$$

or using Dishal's notation

$$Q_u \approx \left(\frac{f_0}{\text{BW}_{3\,\text{dB}}}\right) q_{2,3,\ldots,(n-1)}$$

Table 9.2 Insertion Loss Equations for Low-Loss Butterworth Responses

$$n = 2 \qquad IL = 20 \log \left(\frac{1.414}{Q_0} + 1\right)$$

$$n = 3 \qquad IL = 20 \log \left(\frac{2}{Q_0^2} + \frac{2}{Q_0} + 1\right)$$

$$n = 4 \qquad IL = 20 \log \left(\frac{2.62}{Q_0^3} + \frac{3.41}{Q_0^2} + \frac{2.62}{Q_0} + 1\right)$$

$$n = 5 \qquad IL = 20 \log \left(\frac{3.24}{Q_0^4} + \frac{5.23}{Q_0^3} + \frac{5.23}{Q_0^2} + \frac{3.24}{Q_0} + 1\right)$$

$$n = 6 \qquad IL = 20 \log \left(\frac{3.84}{Q_0^5} + \frac{7.42}{Q_0^4} + \frac{9.11}{Q_0^3} + \frac{7.43}{Q_0^2} + \frac{3.84}{Q_0} + 1\right)$$

$$n = 7 \qquad IL = 20 \log \left(\frac{4.46}{Q_0^6} + \frac{10.0}{Q_0^5} + \frac{14.5}{Q_0^4} + \frac{14.6}{Q_0^3} + \frac{10.0}{Q_0^2} + \frac{4.46}{Q_0} + 1\right)$$

will the resulting bandwidth be equal to the design bandwidth.

For the low Q case, when $Q_u < 10Q_{\min}$, the predistorted k and q values given should be used in order to realize the design bandwidth.

This can be summarized as follows: If $Q_u \geq 10Q_{\min}$, use the infinite Q values of k and q. If $Q_u < 10Q_{\min}$, use the predistorted values. It should be mentioned, for the high-Q case, an insertion loss of 1 dB or less will result. For the low-Q case, the insertion loss of the network is given in the tables, having been defined as

$$IL = 10 \log \frac{P_{\text{out max}}}{P_{\text{out}}} = 10 \log \frac{R_2}{4R_1} \left| \frac{V_{\text{in}}}{V_{\text{out}}} \right|^2 \qquad (9.3.5)$$

where

$$P_{\text{out max}} = \frac{V_{\text{in}}^2}{4R_1}$$

It should be noticed that for most of the predistorted cases, the values of q_1 and q_n are different, and the input and output resonators should be tapped in different points, for the helical filter to be loaded properly.

Example 6

A filter is required with the following specifications:

$f_0 = 30$ Mc

$BW_{3\,dB} = 0.9$ Mc

$BW_{50\,dB} = 4.5$ Mc

$Z_{in} = Z_{out} = 50$ ohms

Insertion loss: 3 dB maximum

Maximum dimensions of filter: $6\frac{3}{8}$ in. by $1\frac{3}{4}$ in. by $2\frac{3}{4}$ in.

Response: Butterworth

SOLUTION

1. Determine the number of resonators. Using Fig. 5.1, with $A_{\max} = 3$ dB, $A_{\min} = 50$ dB, and $\Omega = 4.5/0.9 = 5.0$, we see that between three and four resonators will be required for realization. We must use four resonators.

We could also have used the curve of the Butterworth attenuation characteristics of Chapter 3, to find that a three-pole would only provide 42 dB at the 50-dB point, but the four-pole network satisfies the requirement with 56 dB.

2. Calculate the Q that can be obtained. Since we are using four resonators, five metal thicknesses must be subtracted from the maximum length. For $\frac{1}{16}$ in. walls, this results in approximately 6 in. remaining for four resonators. Each resonator will have

$$S = 6/4 = 1.5 \text{ in.}$$

Then,

$$Q_u = 60S\sqrt{f_0} = 492.9$$

3. Determine if the high-Q case can be realized. For a four-pole Butterworth, $q_{\min} = 2.6$

$$Q_{\min} = 2.6 \frac{30}{0.90} = 86.6$$

Since $Q_u < 10Q_{\min}$, we cannot use the high-Q case and obtain the exact bandwidth.

4. Determine predistorted k and q values and insertion loss. Using the table of 3-dB down k and q values for the Butterworth response, with $n = 4$, and calculating

$$q_0 = \left(\frac{BW_{3\,dB}}{f_0}\right) Q = \frac{0.9}{30} 490 = 14.7$$

We could use the k and q values associated with $q_0 = 13.066$, the closest value to the calculated q_0. It is also possible to plot the values given in the table, as done in Fig. 9.15. Here, the abscissa is

$$d_0 = \frac{1}{q_0}$$

For our example $d_0 = 0.068$, and using the curves of Fig. 9.15 we find

$q_1 = 0.533$

$q_4 = 1.642$

$k_{12} = 1.076$

$k_{23} = 0.554$

$k_{34} = 0.680$

$IL = 1.9$ dB, satisfying the insertion-loss requirement.

5. Computer run (optional). To check the resulting filter, element values can be determined and the filter run on a digital computer to check bandwidth, rejection, and insertion loss. The lumped representation of the helical filter is shown in Fig. 9.16. Figure 9.17 is the computed response.

At the 50-dB rejection points, on the low side 59 dB is achieved, and on the high side the attenuation is 52 dB. The 3-dB bandwidth is 0.9 Mc, as required.

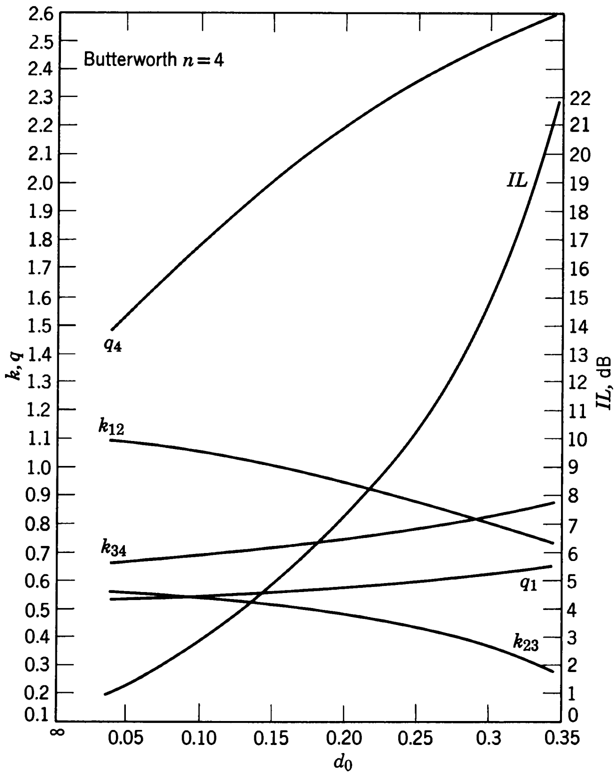


Fig. 9.15. 3-dB down k and q values for Butterworth filter with $n = 4$.

The computed insertion gain is 3.14 dB. The voltage gain due to the impedance step-up is

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \sqrt{\frac{Z_{\text{out}}}{Z_{\text{in}}}} = 1.755$$

This corresponds to 4.88 dB gain. Since the filter's actual insertion gain is 3.14 dB, the insertion loss not considering the impedance step-up is 1.74 dB. This checks closely with the 1.9-dB insertion loss obtained previously, and satisfies the specification of the filter.

6. Compute the resonator dimensions. The necessary design quantities are obtained from Eqs. 9.2.16 through 9.2.21.

$$N = \frac{1600}{f_0 S} = 35.5 \text{ turns}$$

$$n = \frac{1600}{S^2 f_0} = 23.7 \text{ turns per in.}$$

$$Z_0 = \frac{81{,}500}{f_0 S} = 1811.1 \text{ ohms}$$

$$d = 0.66S = 0.99 \text{ in.}$$

$$b = S = 1.5 \text{ in.}$$

$$H = 1.6S = 2.4 \text{ in.}$$

$$d_0 = \frac{1}{2n} = 0.0210 \text{ in., which corresponds to No. 24 copper wire.}$$

7. Compute the shield heights. The dimensions of the coupling shields will now be calculated. We have previously found

$$k_{12} = 1.076$$
$$k_{23} = 0.554$$
$$k_{34} = 0.680$$

Now

$$K = k\,\frac{\text{BW}_{3\,\text{dB}}}{f_0}$$

$$K = 30 \times 10^{-3} k$$

and

$$K_{12} = 32.3 \times 10^{-3}$$
$$K_{23} = 16.6 \times 10^{-3}$$
$$K_{34} = 20.4 \times 10^{-3}$$

From Fig. 9.13,

$$\frac{h_{12}}{d} = 0.66$$

$$\frac{h_{23}}{d} = 0.46$$

$$\frac{h_{34}}{d} = 0.52$$

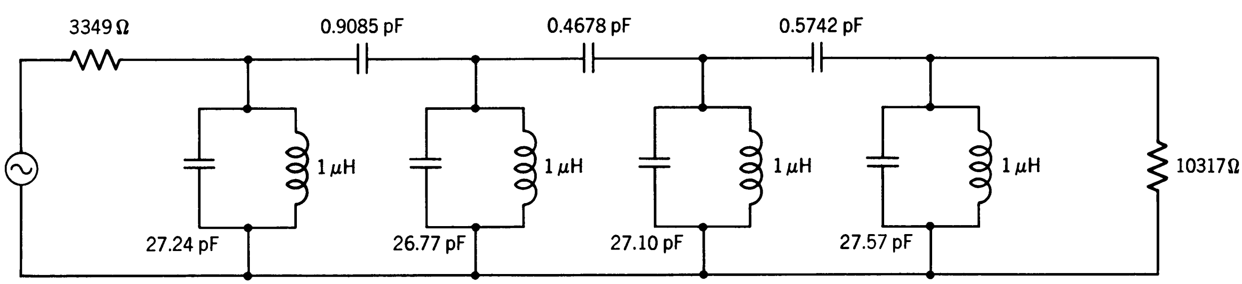


Fig. 9.16. Lumped representation of helical filter of Example 6.

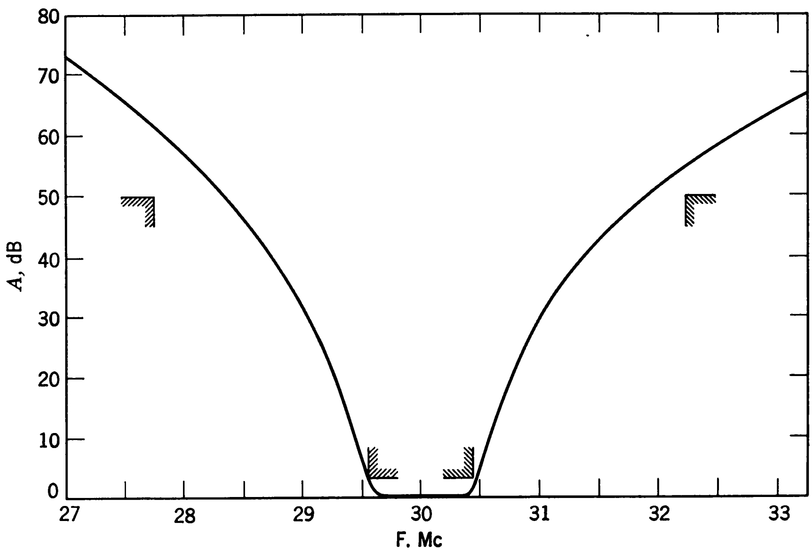


Fig. 9.17. Computed response of network of Fig. 9.16.

We have calculated $d = 0.99$ inches. Therefore,

$$h_{12} = 0.653 \text{ in.}$$

$$h_{23} = 0.455 \text{ in.}$$

$$h_{34} = 0.515 \text{ in.}$$

We will use $\frac{1}{16}$ in. thick material for the shields. As mentioned earlier, the correction factor is 1.075. So,

$$h_{12} = 0.702 \text{ in.}$$

$$h_{23} = 0.489 \text{ in.}$$

$$h_{34} = 0.554 \text{ in.}$$

This is the opening between adjacent helices, and may be at the top or bottom of the resonator.

8. Calculate the input and output tap. We must determine the position of the input and output tap to match a 50-ohm source and load. We have obtained and calculated the following information.

$$q_1 = 0.533$$

$$q_4 = 1.642$$

$$Q_u = 490$$

$$Z_0 = 1811.1 \text{ ohms}$$

$$N = 35.5 \text{ turns}$$

Using Eqs. 9.3.1, 9.3.2, and 9.3.3 for the input coil,

$$Q_d = \tfrac{1}{2}q_1 \frac{f_0}{\text{BW}_{3\,\text{dB}}} = 8.9$$

$$\frac{R_b}{Z_0} = \frac{\pi}{4}\left(\frac{1}{Q_d} - \frac{1}{Q_u}\right) = 0.0866$$

$$\sin\theta = \sqrt{\frac{R_b}{2Z_0}\frac{R_{\text{tap}}}{Z_0}} = 0.0346$$

$$\theta = 1.98^\circ$$

$$\text{tap} = \frac{N\theta}{90^\circ} = 0.78 \text{ turn from ground.}$$

For the output coil the resulting tap position is approximately 0.44 turn from ground.

9. Determine the final outside dimensions of the can. Finally, the dimensions of the can, considering $\frac{1}{16}$ in. thick metal thickness is

$$\text{Length} = 4S + (5 \times \tfrac{1}{16}) = 6.313 \text{ in.}$$

$$\text{Width} = S + (2 \times \tfrac{1}{16}) = 1.625 \text{ in.}$$

$$\text{Height} = H + (2 \times \tfrac{1}{16}) = 2.525 \text{ in.}$$

10. Tuning the filter. The necessary design information has now been obtained. Tuning of the filter is accomplished by use of a sweep generator, to be described in Section 9.4.

9.4 ALIGNMENT OF HELICAL FILTERS

When a filter is designed and constructed to the best of the engineering ability, the next very important problem is to adjust to the specific response or "tuning up". The method described is known as Dishal's method. Filter constants can always be reduced to only three fundamental types:

1. f_0—the resonant frequency of each resonator
2. $d_i = 1/Q_i$—the decrement of the ith resonator, defined as the fractional bandwidth between the 3-dB down points, when the resonator is considered separately
3. $K_{i(i+1)}$—the coefficient of coupling between the ith and the $(i+1)$th resonator. Here, $K_{i(i+1)} = (\Delta f/f_0)k_{i(i+1)}$.

All of the required values for K and Q are given for an n-resonant circuit filter that will produce the response

$$\left(\frac{V_p}{V}\right)^2 = 1 + \left(\frac{\Delta f}{\Delta f_{3\,\mathrm{dB}}}\right)^{2n}$$

where V_p is the voltage output at the peak of the response curve in the passband.

Most selective circuit designs incorporate a trimming adjustment for setting the resonant frequency of each resonator, such as a tuning screw for the helical resonator filter. The coefficient of coupling between adjacent resonators may be variable, and a method is given for easily adjusting the exact desired value.

In this tuning method, the filter is completely assembled, and attention is concentrated on amplitude phenomena occurring in the first resonator of the filter at the desired resonant frequency. The alignment procedure will be described using the four-resonator bandpass configuration shown in Fig. 9.18. The procedure is applicable to all coupled-resonant circuit filters, whether they be low frequency, high-frequency, VHF, microwave-frequency or even waveguide filters. The steps of the procedure are as follows:

1. Connect the generator to the first resonator of the filter and the load to the last resonator of the filter in exactly the same manner as they will be connected in actual use.
2. Couple a nonresonant detector directly and very loosely to either the electric or the magnetic field of the first resonator of the filter. A nonresonant detector may be said to be "very loosely" coupled when it lowers the unloaded Q of the resonator by less than 5%.
3. Completely detune all resonators. A resonator is sufficiently detuned when its resonant frequency is at least 10 passband widths away from passband midfrequency.
4. Set the generator frequency to the desired midfrequency of the filter.
5. Tune resonator 1 for maximum output indication on the detector. Lock the tuning adjustment.
6. Tune resonator 2 for minimum output indication on the detector. Lock the tuning adjustment.
7. Tune resonator 3 for maximum output and lock the tuning adjustment.
8. Tune resonator 4 for minimum output and lock the tuning adjustment.

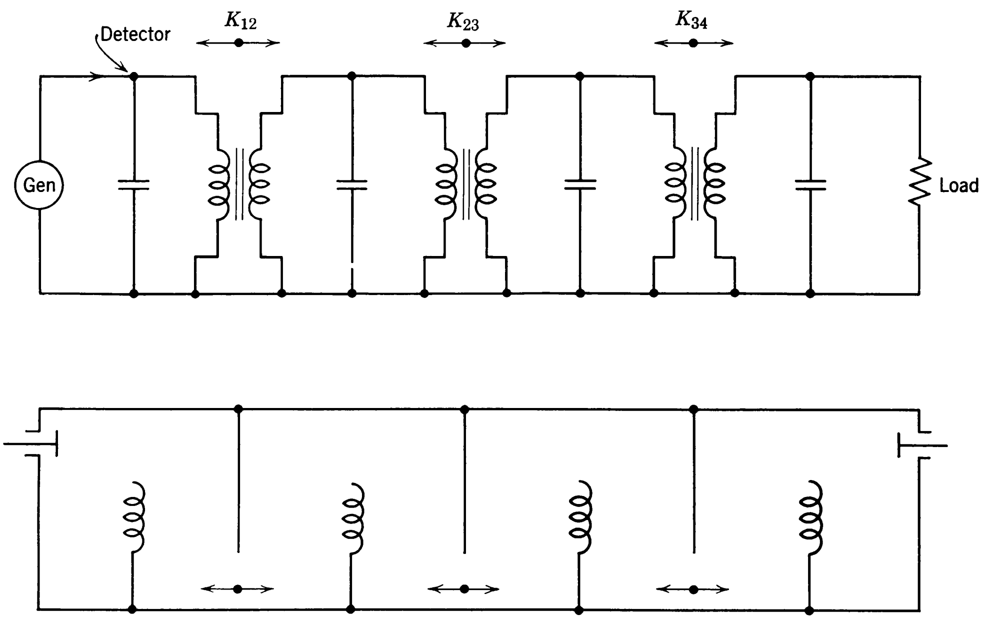


Fig. 9.18. Four-pole conventional helical network used to illustrate alignment procedure.

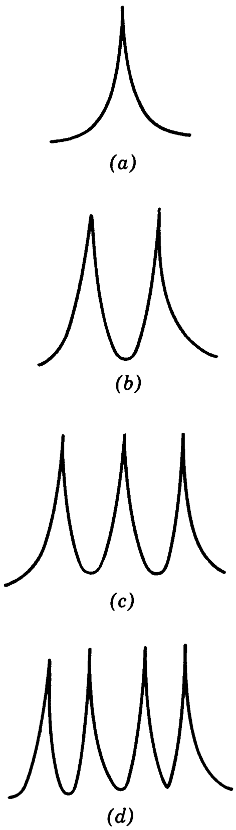


Fig. 9.19. Reproductions of oscillograms of the amplitude frequency phenomenon occurring in resonator 1 as alignment steps are performed.

The alignment of the filter is now complete. Figure 9.19 shows the amplitude-frequency phenomena occurring in resonator 1 as the alignment steps are performed. They may be observed on an oscilloscope, as the filter is swept. It should be realized that since the alignment adjustments depend exclusively on the amplitude of the response at f_0, a sweep-frequency generator is not required, and all adjustments can be made with a single-frequency input, f_0.

Figure 9.19*a* is the oscillogram produced when resonator 2 is detuned or short circuited, and the input resonator 1 is adjusted for a maximum signal at f_0. Figure 9.19*b* occurs when the second resonator is tuned for minimum amplitude at f_0. The third resonator is detuned. Figures 9.19*c* and 9.19*d* show the continuation of the procedure as detected in resonator 1.

It can be seen that when the ith resonator is tuned, there will be i peaks and $i - 1$ valleys produced in resonator 1. It should be remembered that if it is impossible to completely detune all resonators, a single device may be used to short circuit the resonator immediately following the one being tuned. This short must be effective at the frequency involved.

Measurement of Coupling

The fundamental procedure is based upon the consideration that every pair of adjacent resonators is a double-tuned—that is, a two-pole circuit (with all the other resonators completely detuned). In a double-tuned circuit with Q_A and Q_B equal to infinity, the fractional bandwidth between primary response peaks is exactly equal to the coefficient of coupling between resonators A and B. This direct relationship makes this phenomena an excellent one to use as the basis of an experimental procedure for adjusting of coefficient coupling to a desired value.

When the unloaded Q of the resonators are very high, but Q is not infinity, the curve of Fig. 9.20 supplies a way of finding the exact coupling between adjacent resonators. To determine the amount of coupling between adjacent resonators, the following steps must be taken:

1. Designate as A the lower Q resonator and B the higher Q resonator.
2. Couple a nonresonant signal generator directly and very loosely to either the electric or magnetic field of resonator A.
3. Couple a nonresonant detector directly and loosely to either the electric or magnetic field of resonator A.
4. Completely detune all the resonators in the filter.
5. Tune resonator A for maximum output from the detector. Record the signal generator input and the detector output.
6. Tune resonator B for minimum output from the detector (as in alignment procedure in the previous section). Increase the signal generator input to produce the same output obtained in Step 5 (see Fig. 9.21).
7. The ratio of the signal generator input in Step 6 to that in Step 5 is the abscissa in Fig. 9.20. From the ordinate of this graph, read the ratio of coupling K between resonators A and B to the percentage bandwidth $\Delta f_p/f_0$ between the amplitude peaks that are now present across resonator A.
8. Carefully measure the bandwidth Δf_p between the response peaks of resonator A.
9. The exact coefficient of coupling is equal to the fractional bandwidth between these peaks time the ordinate value obtained in Step 7.

For the low-Q case, in a double-tuned circuit with Q_A and Q_B only two or three times the fractional bandwidth, the coefficient of coupling required in the

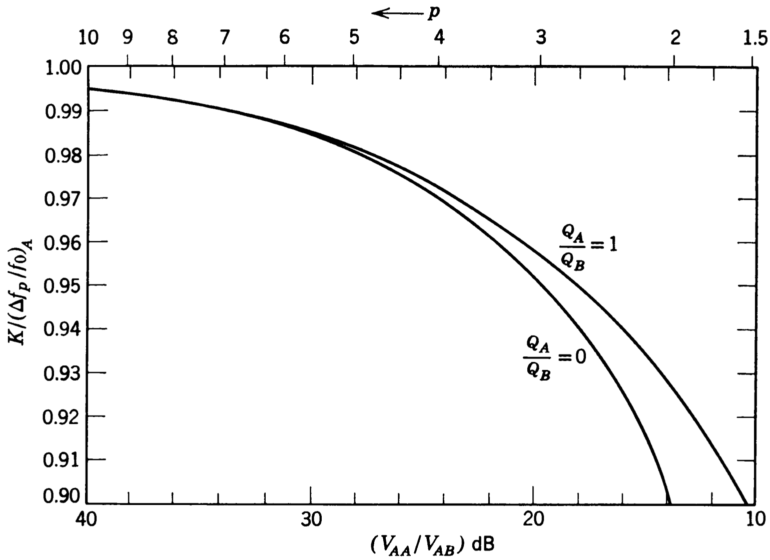


Fig. 9.20. Method of finding exact coefficient of coupling K between two resonators, when the K required is much greater than the unloaded decrements.

network to produce desirable response shapes are not much greater than the unloaded decrements. The resulting peak to valley ratio will not be very great when the previous procedure is used, and the peaks will not be sharply defined. For this case, the coefficient of coupling can be found in terms of the measured Q of the resonators being used.

In order to determine the amount of coupling between adjacent resonators for the low-Q case, the following steps must be taken:

(Repeat steps 1 through 6, as for the high Q case.)

7. The ratio of signal generator input of Step 6 to that in Step 5 is the abscissa of the graph of Fig. 9.22. From the ordinate of this graph read the ratio of the coefficient of the coupling K_{AB} to the geometric mean of the decrement of the resonators A and B.

8. Carefully measure the Q of each resonator A and B by measuring the 3-dB bandwidth. See Eq. 9.4.7 and related discussion.

9. The exact coefficient of coupling is equal to the geometric mean of A and B decrement times the ordinate value in Step 7.

Adjustment of Coupling in a Helical Filter

It is often desirable to be able to set or to check each coefficient of coupling of a filter without going through the procedure of converting into a double-tuned circuit. This can be accomplished by measurements made entirely in the input resonator.

There are, in practice, two cases which must be considered. In the first case, the unloaded Q's of the resonators being used are very much greater than the fractional midfrequency $f_0/\mathrm{BW}_{3\,\mathrm{dB}}$ being used—that is, the unloaded individual Q's are essentially infinite. In the second case, the unloaded Q's of the resonators are only four, or five, or fewer times $f_0/\mathrm{BW}_{3\,\mathrm{dB}}$.

For the first case above, which is the most common case in helical filtering, the K's can be approximately measured, in consecutive order, by measuring the

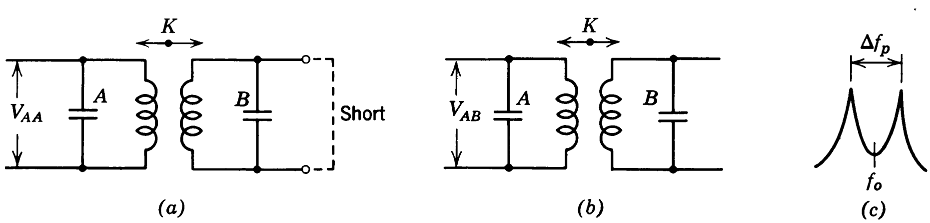


Fig. 9.21. Procedure for obtaining coefficient of coupling K between two resonators when the K required is much greater than the unloaded decrement. (a) Resonate circuit A for maximum V_{AA} (B shorted). (b) Resonate circuit B for minimum V_{AB} (short removed). (c) Bandwidth between primary peaks of V_{AB}.

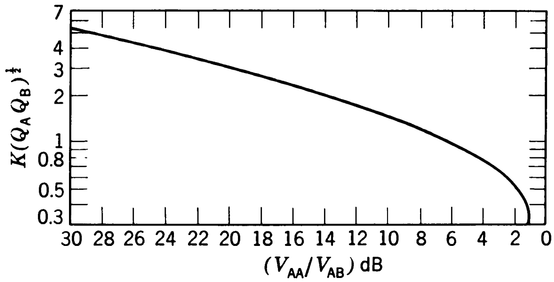


Fig. 9.22. Method of obtaining exact coefficient of coupling K between two resonators when the K required is not much greater than the unloaded decrements.

bandwidth between the various response peaks appearing in resonator 1, as each of the following resonators is resonated in consecutive order. If the unloaded Q's are approximately 100 times $f_0/\mathrm{BW}_{3\,\mathrm{dB}}$, then this procedure gives exact answers. It should be remembered that there will be i response peaks occurring in the input resonator, when the ith resonator is correctly tuned.

The first step is to be sure that the input resonator is correctly loaded by adjusting the position of the tap. The 3-dB bandwidth of the response of the first resonator is given by

$$\Delta_{3\,\mathrm{dB}} = \frac{\mathrm{BW}_{3\,\mathrm{dB}}}{q_1} \tag{9.4.1}$$

when the following resonator is shorted. Here, the value of q_1 is obtained from the tables of 3 dB down k and q values of Chapter 6, and $\mathrm{BW}_{3\,\mathrm{dB}}$ is the desired filter bandwidth.

Next, in tuning the completed filter, we must calculate the frequencies of the peaks obtained as the filter is tuned resonator by resonator. This is done by using Eqs. 9.4.2 through 9.4.5. The values of k, are obtained from the tables of Chapter 6, and $\mathrm{BW}_{3\,\mathrm{dB}}$ is again the desired filter 3-dB bandwidth.

The resulting Δf_p is the distance between amplitude response peaks occurring in the first resonator, and by adding to and subtracting from f_0 the quantities $\Delta f_p/2$, the absolute frequencies of the peaks are obtained.

In setting the first resonator,

$$\Delta f_p = 0 \tag{9.4.2}$$

Here, one peak occurring at f_0 is obtained and the 3-dB bandwidth is given by Eq. 9.4.1. For the second resonator,

$$\Delta f_p = k_{12}\mathrm{BW}_{3\,\mathrm{dB}} \tag{9.4.3}$$

For the third resonator,

$$\left(\frac{\Delta f_p}{\mathrm{BW}_{3\,\mathrm{dB}}}\right)^3 - (k_{12}^2 + k_{23}^2)\frac{\Delta f_p}{\mathrm{BW}_{3\,\mathrm{dB}}} = 0$$

yields two solutions:

$$\Delta f_p = 0$$

and

$$\Delta f_p = \sqrt{k_{12}^2 + k_{23}^2}\,\mathrm{BW}_{3\,\mathrm{dB}} \tag{9.4.4}$$

This indicates that three peaks are obtained, one at f_0, and a lower and higher peak separated by the distance Δf_p.

For the fourth resonator,

$$\left(\frac{\Delta f_p}{\mathrm{BW}_{3\,\mathrm{dB}}}\right)^4 - (k_{12}^2 + k_{23}^2 + k_{34}^2) \times \left(\frac{\Delta f_p}{\mathrm{BW}_{3\,\mathrm{dB}}}\right)^2 + (k_{12}k_{34})^2 = 0$$

again yields two solutions:

$$\Delta f_p = \mathrm{BW}_{3\,\mathrm{dB}}\left(\frac{(k_{12}^2 + k_{23}^2 + k_{34}^2) \pm \sqrt{(k_{12}^2 + k_{23}^2 + k_{34}^2)^2 - 4(k_{12}k_{34})^2}}{2}\right)^{1/2} \tag{9.4.5}$$

Here, four peaks are obtained; the larger value of Δf_p is the distance between the outer pair, the smaller value of Δf_p is the distance between the inner pair.

With Eqs. 9.4.2 through 9.4.5, filters up to and including seven poles may be completely tuned. This is done by first working from input to output and then turning the filter around, working from output to input. Figure 9.23 lists values of normalized $\Delta f_p/\mathrm{BW}_{3\,\mathrm{dB}}$ for the Butterworth response shape for 2-, 3-, 4-, 5-, 6-, and 7-pole filters, as well as values of $1/q_1 = \Delta_{3\,\mathrm{dB}}/\mathrm{BW}_{3\,\mathrm{dB}}$ for the purpose of adjusting the input and output tap.

For the second case described—that is, where the unloaded Q is only four or five times $f_0/\mathrm{BW}_{3\,\mathrm{dB}}$, the coefficients of coupling should be set or measured in consecutive order as follows: Accurately measure the Q of each resonator in the filter, then precede step by step, through the alignment procedure, accurately measuring and recording the magnitudes of the maxima and minima produced.

The ratio of the detector output obtained when resonator 1 is alone resonated to that obtained when resonator i is resonated is given by

$$\left(\frac{V_{1,1}}{V_{1,i}}\right) = \left[1 + \cfrac{p_{12}^2}{1 + \cfrac{p_{23}^2}{1 + \cfrac{\cdots}{1 + p_{(i-1)i}^2}}}\right] \tag{9.4.6}$$

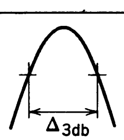
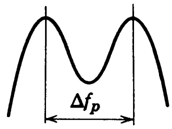
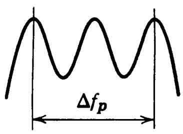
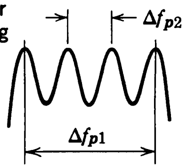
Number of resonators			2	3	4	5	6	7
Tuning resonator no. 1, and setting tap.		$\dfrac{\Delta_{3db}}{BW_{3db}} =$	0.707	1.000	1.305	1.618	1.931	2.247
Tuning resonator no. 2, and setting K_{12}		$\dfrac{\Delta f_p}{BW_{3db}} =$	0.707	0.707	0.840	1.000	1.170	1.340
Tuning resonator no. 3, and setting K_{23}		$\dfrac{\Delta f_p}{BW_{3db}} =$		1.000	1.000	1.144	1.318	1.498
Tuning resonator no. 4, and setting K_{34}		$\dfrac{\Delta f_{p1}}{BW_{3db}} =$			1.154	1.182	1.342	1.518
		$\dfrac{\Delta f_{p2}}{BW_{3db}} =$			0.612	0.470	0.452	0.466

Fig. 9.23. Normalized Δf_p and Δ_{3dB} for Butterworth filters.

where $p_{12}^2 = K_{12}^2 Q_1 Q_2$, and so on, and

$$K_{12} = \frac{\Delta f}{f_0} k_{12}$$

Since we know the desired value of K, and have measured each Q, we can calculate the required $V_{1,1}/V_{1,i}$ ratio from Eq. 9.4.6, and compare to the measured value. The most trustworthy method of making accurate loaded or unloaded Q measurements on a resonator that is part of the filter chain seems to be as follows:

1. Completely assemble the filter.
2. Completely detune all resonators except the one to be measured. Obviously complete detuning of the resonator on each side of the one being measured should be satisfactory.
3. A nonresonant signal generator is coupled directly and very loosely to either the electrical or magnetic field of the resonator.
4. A nonresonant detector is coupled very loosely and preferably to the field opposite to that being used for the generator. In other words, make sure that there is negligible direct coupling between generator and detector.
5. Using an unmodulated wave, or an amplitude-modulated wave checked for negligible frequency modulation from the signal generator, measure the frequency difference Δf_β between the points that are V_p/V_β down from the peak response. The resonator Q is given by

$$Q = \frac{f_0}{\Delta f_\beta} \sqrt{\left(\frac{V_p}{V_\beta}\right)^2 - 1} \tag{9.4.7}$$

When high Q's are to be measured, the setup must be capable of measuring very small-percentage bandwidths.

Sweep Display Setups

Figure 9.24 illustrates a typical sweep display setup for viewing the transmission characteristics of a VHF filter. To view the impedance characteristics of the filter, the block diagram of Fig. 9.25 can be used. Finally, Fig. 9.26 shows a setup for tuning the filter by means of a small probe placed in the first resonant cavity. The signal from this probe must be approximately 20-dB down, in order not to load down the first resonator.

9.5 EXAMPLES OF HELICAL FILTERING

The filter of Fig. 9.14 is a 540-Mc helical filter which is tuned in order to provide an equiripple group

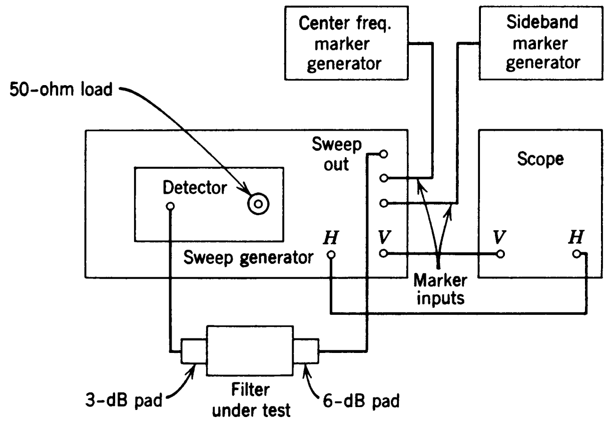


Fig. 9.24. Typical sweep display setup of transmission characteristics of a filter at VHF.

delay. The can of Fig. 9.27 is used for two filters, with center frequencies at 28 and 32 Mc. The length is approximately 4 inches. The filters have 3-dB bandwidths of 1.5 Mc and an insertion loss of 1.4 dB was obtained. The coils have approximately 65 turns. Tap coupling was used for the input and output coils, providing an impedance of 75 ohms. One unusual specification for these filters was that at 30 Mc, 29 dB $\pm$ 0.5 dB was to be provided for both filters.

When a high value of attenuation is needed at frequencies near $3f_0$, the helical filter must be followed by a lowpass *LC* filter, since helical filters have a secondary or spurious response at this frequency. Figure 9.28 shows a VHF helical filter–lowpass combination. A minimum attenuation of 60-dB is achieved at frequencies up to 6.6 times f_0.

The design shown in Fig. 9.29 is actually two filters in one case. The helical section is a 30-Mc bandpass

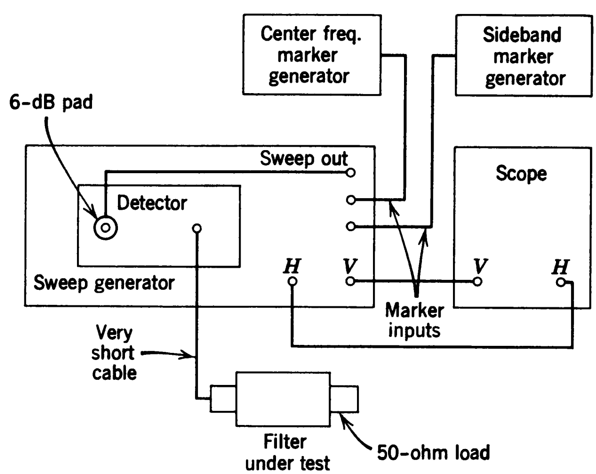


Fig. 9.25. Typical sweep display setup of impedance characteristic of a filter at VHF.

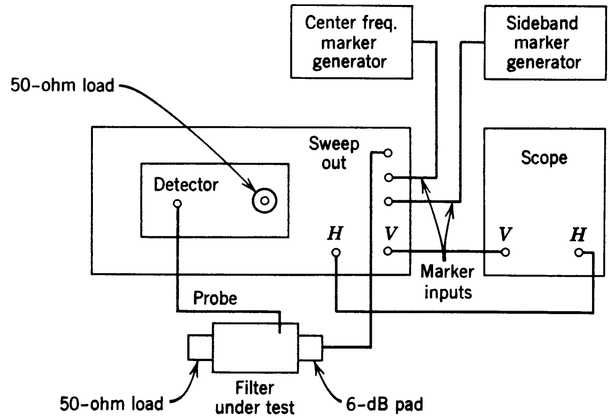


Fig. 9.26. Typical sweep display setup of amplitude phenomena in first cavity for VHF filters.

filter with a 3-dB bandwidth of 330 kc, and the filter mounted on the vertical printed circuit board is a 1 Mc wide *LC* filter at the same center frequency. Both filters are tuned to provide a Gaussian response. This design is unusual because of the small physical size (1 $\times$ 1.5 $\times$ 2 inches), made possible with the square layout of the helical resonators.

Use of Helical Filters in Parametric Multipliers

For frequency multipliers, a varactor diode is usually inserted between bandpass filters. The first filter is designed for the fundamental signal to be multiplied, and the output filter is designed for the specific harmonic in which the fundamental frequency is transformed by the parametric diode. The entire network from input to output is still considered as a filter, because the nonlinear diode is absorbed by the adjacent elements in such a fashion that the low-frequency filter presents an appropriate load, and to the harmonic filter, the diode represents a nonlinear capacitor providing a source of harmonic frequencies with high impedance.

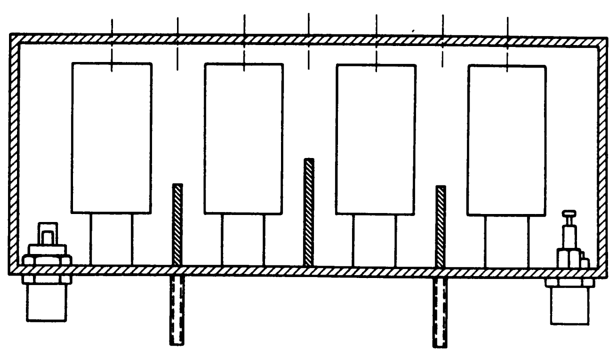


Fig. 9.27. Four resonator 28- and 32-Mc filter.

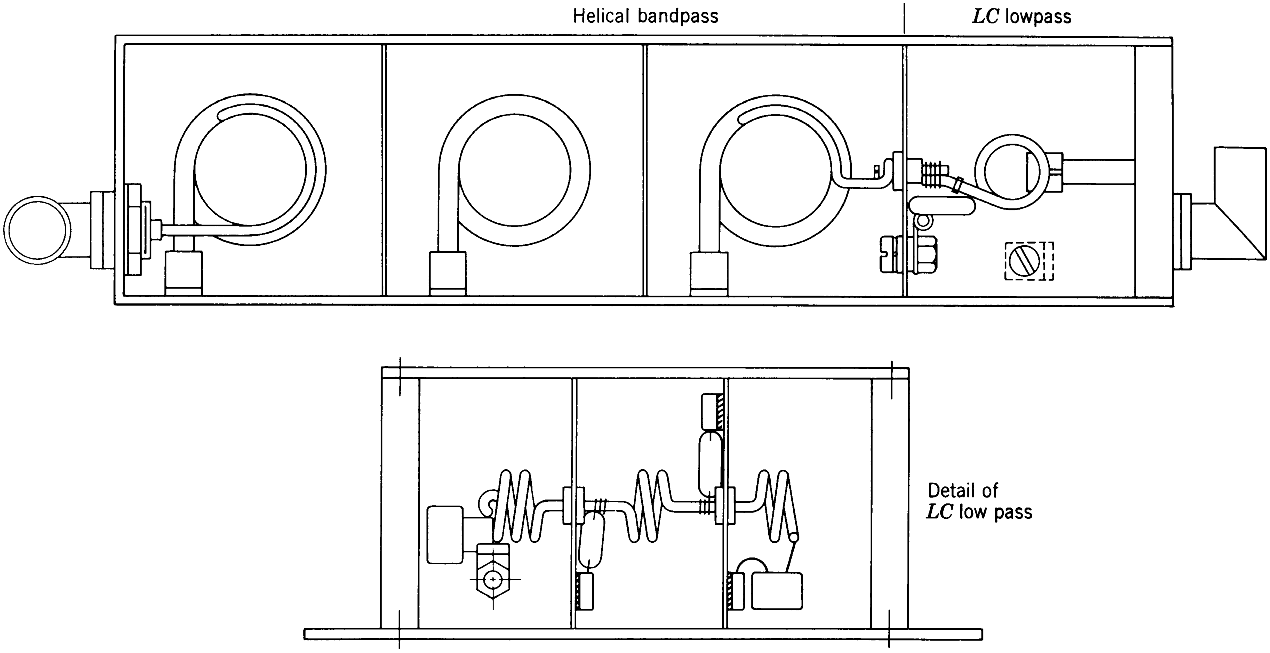


Fig. 9.28. Helical bandpass and *LC* lowpass filter combination.

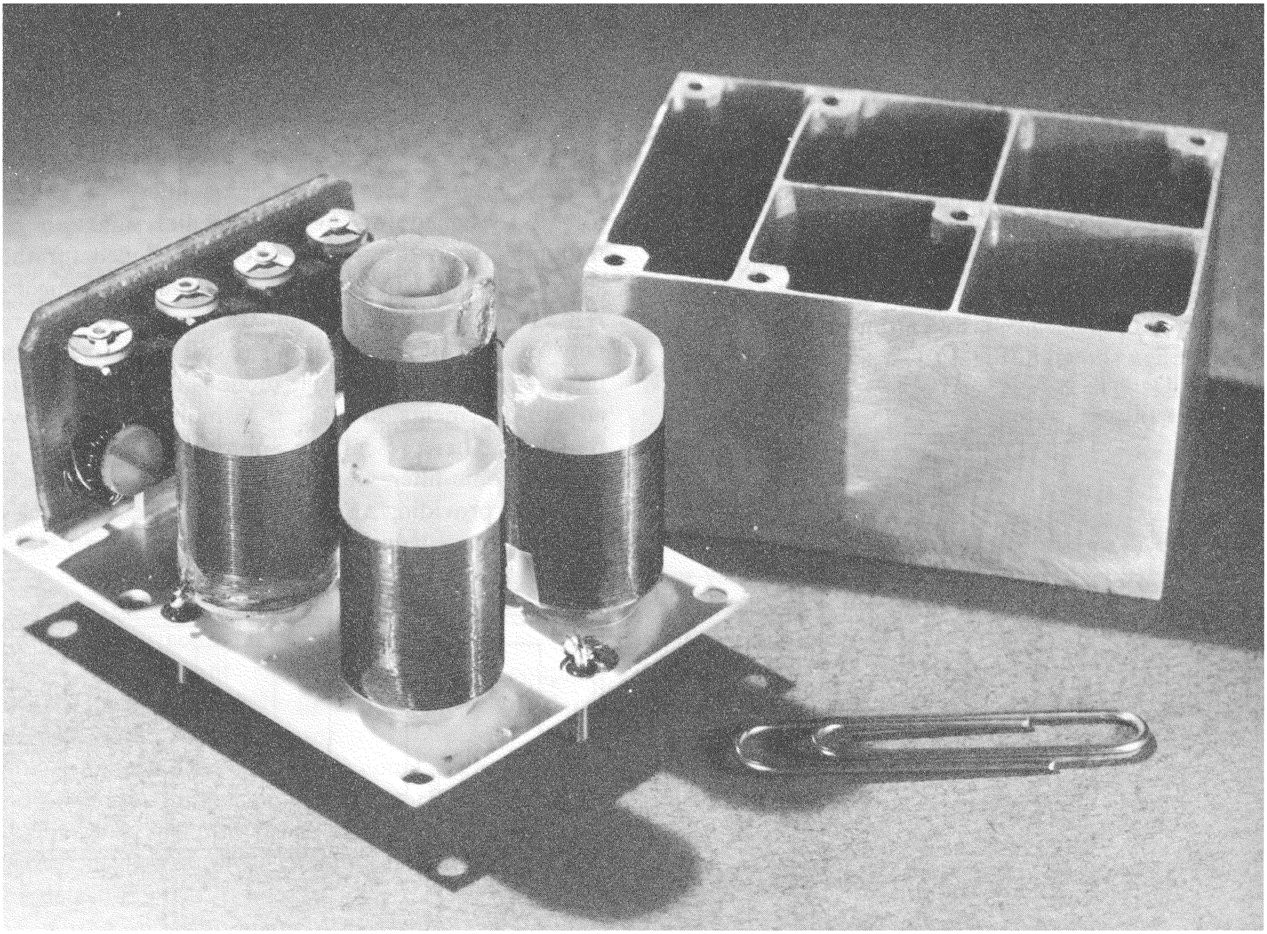


Fig. 9.29. 30-Mc helical and *LC* filter.

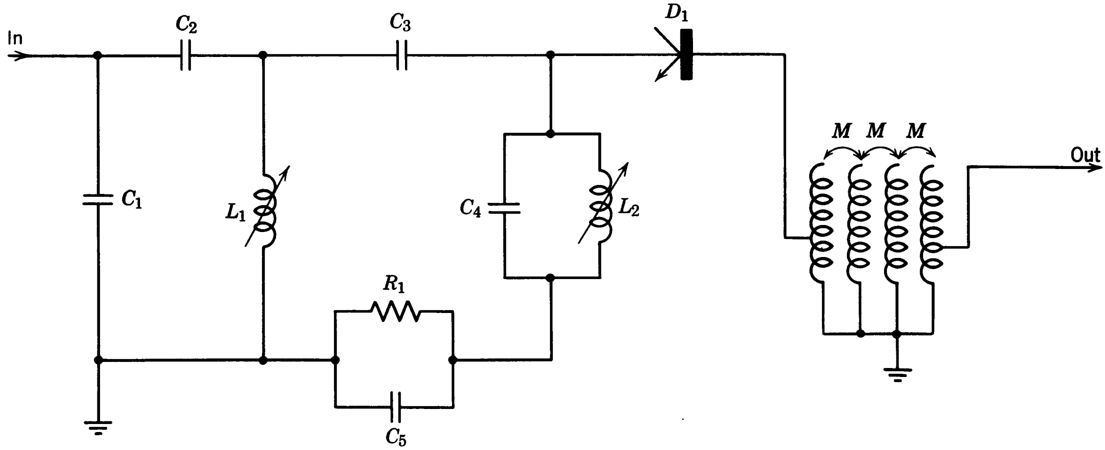


Fig. 9.30. Schematic diagram of 63 Mc × 6 = 378 Mc parametric multiplier.

The whole assembly can be designed in such a way that no spurious modes from the varactor will be passed through the high-frequency portion of the network. The fundamental frequency filter, in many cases can be a conventional *LC* network. The output from the varactor is connected to the input tap of the helical resonator filter. A schematic diagram of such a parametric multiplier is shown in Fig. 9.30. To eliminate unwanted outputs, the helical filter consists of several cavities. The high-quality helical filter adds only a negligible amount of loss to the multiplier network, in comparison to a conventional *LC* filter. A narrow passband is usually used since any noise in the initial stage of the harmonic multiplier or synthesizer is very damaging for the following stages at microwave frequencies. This arrangement with helical filters will cover the entire VHF and UHF bands, providing the cleanest harmonic output and the cleanest source of the above mentioned frequencies.